Quantum Computing Bootcamp Assignment-1

1. Study the properties of a vector space and argue whether polynomials form a vector space or not, and why?

Ans: A vector space is a nonempty set V of objects, called vectors, which are defined by two operations, called addition and multiplication by scalars (real numbers), subject to the ten axioms below. The axioms must hold for all u, v, and w in V and all scalars c and d.

1. u + v is in V
2. u+v = v+u
3. (u+v)+w=u+(v+w)
4. There is a vector (called the zero vector) 0 in V such that u+0=u
5. For each u in V, there is a vector u in V satisfying u + (u) =0
6. cu is in V
7. c(u+v) = cu+cv
8. (c +d) u = cu+du
9. (cd)u = c(du)
10. 1u = u

Let n ≥ 0 be an integer and let

Pn = the set of all polynomials of degree at most n≥ 0

Members of Pn have the form

p(t) = a0 +a1t +a2t2 +…. +antn

, where a0, a1, ……, an are real numbers and t is a real variable. The set Pn is a vector space.

We will verify 3 out of the 10 axioms here. Let p(t) = a0 +a1t +a2t2 +…. +antnand

q(t) = b0 +b1t +b2t2 +…. +bntn. Let c be a scalar.

**Axiom 1:** The polynomial p +q is defined as follows: (p +q)(t) = p(t)+q(t)

Therefore,

(p +q)(t) = p(t)+q(t)

=( …… ) +( …… ) t + … +( …… ) tn

which is also a (…… ) of degree at most …….So p+q is in Pn.

**Axiom 4:**

0 =0+0t + … +0tn

(zero vector in Pn)

(p +0)(t)= p(t)+0 = (a0 +0)+(a1 +0)t + … +(an +0)tn

=a0 +a1t + … +antn =p(t)

and so p+0 =p

**Axiom 6:**

(cp)(t) = cp(t) = ( …… ) + ( ……)t+ … + ( …… )tn which is in Pn.

The other 7 axioms also hold, so Pn is a vector space.

Thus, under scalar multiplication and addition, polynomials form a vector space. Even if an infinite-dimensional space of all polynomials has infinite basis vectors, it is still a vector space.

Reference:

* [Math 2331 – Linear Algebra - 4.1 Vector Spaces & Subspaces](https://www.math.uh.edu/~jiwenhe/math2331/lectures/sec4_1.pdf)